Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name: Complex Analysis – I

Subject Code:5SC01MTC3 **Branch :M.Sc.(Mathematics)**

Semester : 1 **Date : 04/12/2015** **Time :10:30 To 1:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Prove that $z^2 + \bar{z}^2 = 2$represents a hyperbola. **(02)**

b. If $f(z)$ is real and analytic in domain then prove that f is constant. **(02)**

c. Prove that $\sin h(iz) = i \sin z$. **(01)**

d. State polar form of C –R equation. **(01)**

e. Find logarithmic value of i. **(01)**

Q-2 **Attempt all questions** **(14)**

a) Suppose $f(z) = u + iv, z_0 = x_0 + iy_0$ and $w_0 = u_0 + iv_0$, then prove that $\lim_{z \to z_0} f(z) = w_0$ if and only if $\lim_{(x,y)\to(x_0,y_0)} u(x,y) = u_0$ and $\lim_{(x,y)\to(x_0,y_0)} v(x,y) = v_0$ **(06)**

b) Let $\mathbb{C}$ be the collection of all complex numbers. Prove that $(\mathbb{C}, +)$ is an abelian group **(04)**

c) Prove that $(1 + i\sqrt{3})^n + (1 - i\sqrt{3})^n = 2^{n+1} \cos(\frac{n\pi}{3})$. **(04)**

OR

Q-2 **Attempt all questions** **(14)**

a) Suppose f is differentiable at z_0 and g is differentiable at $f(z_0)$. Then prove that gof is differentiable at z_0 and $\frac{d}{dz}[gof(z)] = g^{'}(f(z_0)f^{'}(z_0)$. **(06)**

b) Prove that composition of continuous function is continuous. **(04)**

c) Prove that $f(z) = |z|^2$ is differentiable only at origin. **(04)**

Q-3 **Attempt all questions** **(14)**

a) State and prove C – R equation for an analytic function. **(06)**

b) Show that $u(x,y) = 2x - x^3 + 3xy^2$ is harmonic. Find harmonic conjugate of $u(x,y)$. **(04)**

c) Prove that $e^{\bar{z}}$ is nowhere differentiable. **(04)**

OR

Q-3 a) State and prove sufficient condition for a function to be an analytic function. **(06)**

b) Prove that $sin^{-1}z = -i\log(iz + \sqrt{1-z^2})$. **(04)**

c) Find all possible roots of $(-8 - i\,8\sqrt{3})^{\frac{1}{4}}$ **(04)**

SECTION – II

Q-4 **Attempt the Following questions** **(07)**

a. If c be the curve $c: z(t) = 1 - 3it, t \in [-1,1]$ where $x(t) = 1$ $y(t) = -3t$then find arc length of the curve. **(02)**

b. If $z_n \to z$ then prove that $|z_n| \to |z|$. **(02)**

c. Write Maclaurin's series of $\cos z$. **(01)**

d. Give an example of removable singularity. **(01)**

e. Obtain residue of $f(z) = \frac{9z+i}{z\,(z^2+1)}$ at $z = i$. **(01)**

Q-5 **Attempt all questions** **(14)**

a) Evaluate: $\int_{-\infty}^{\infty} \frac{dx}{(x+1)(x^2+2)}$. **(05)**

b) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent's series for (a) $|z| < 1$ (b) $1 < |z| < 3$. **(05)**

c) Evaluate $\int_c \frac{z+2}{z}\,dz$ where c is the lower half of $z = 2e^{i\theta}$. **(04)**

OR

Q-5 a) Find upper bound for the absolute value of the integral $\int_c \frac{z+4}{z^3-1}\,dz$ where $c: |z| = 2$from$z = 2$ to $z = 2i$ that lies in first quadrant. **(05)**

b) State and prove Residue theorem. **(05)**

c) Evaluate $\int_c \frac{dz}{z^2+9}$ where c is (a) $|z - 3i| = 4$ (b) $|z| = 5$. **(04)**

Q-6 **Attempt all questions** **(14)**

a) Let c be a simple closed contour. Suppose f is analytic function within and on c. If z_0 is a point interior to c then prove that $f'(z_0) = \frac{1}{2\pi i}\int_c \frac{f(z)}{(z-z_0)^2}\,dz$. **(07)**

b) State and prove Taylor's theorem. **(07)**

OR

Q-6 **Attempt all Questions**

a) State and prove Cauchy's inequality and deduce Liouville's theorem. **(07)**

b) If four points z_1, z_2, z_3, z_4 of the $z-$ plane map on to the points w_1, w_2, w_3, w_4 of the W −plane respectively under the bilinear transformation then prove that $\frac{(w_1-w_2)(w_3-w_4)}{(w_1-w_4)(w_3-w_2)} = \frac{(z_1-z_2)(z_3-z_4)}{(z_1-z_4)(z_3-z_2)}$. Also Find bilinear transformation that maps the point $z_1 = 0, z_2 = 1, z_3 = \infty$ on to $w_1 = -1, w_2 = -i, w_3 = 1$ respectively. **(07)**